PROBLEM 1

Calculate the integral $\iint_S \hat{e}_\varphi \times \hat{n} dS$

where S is: $x^2 + y^2 + z^2 = 1 \qquad x, y, z \geq 0$

$\hat{n} \cdot \hat{e}_z \leq 0$

PROBLEM 2

Show that the line integral $\int_P^Q \left(\frac{1}{r}\hat{e}_r + \frac{1}{r\sin\theta}\hat{e}_\varphi \right) \cdot d\overline{r}$

from point P: $(r,\theta,\varphi) = \left(1, \frac{\pi}{2}, \frac{\pi}{2}\right)$

till point Q: $(r,\theta,\varphi) = \left(3, \frac{\pi}{4}, \frac{3\pi}{2}\right)$

is independent from the integration path. For simplicity, assume that the path does not intersect the plane $\varphi=\pi$ neither the z-axis.

Calculate the value of the line integral.

PROBLEM 3

Consider the following vector field:

$$\bar{A} = \frac{1}{r^3}\left(\cos 2\theta \hat{e}_\theta - \sin 2\theta \hat{e}_r\right)$$

(a) Calculate $rot\,\bar{A}$
(b) Calculate $div\,\bar{A}$
(c) Has the vector field a potential: $\bar{A} = grad\,\psi$?
Justify the answer and (if positive) calculate ψ

PROBLEM 4

Calculate $\nabla^2 \hat{e}_\varphi$ where $\hat{e}_\varphi$ is a base vector in cylindrical coordinates.

Use the following expression : $\nabla^2 \hat{e}_\varphi = \nabla\left(\nabla \cdot \hat{e}_\varphi\right) - \nabla \times \left(\nabla \times \hat{e}_\varphi\right)$